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EFFECT OF VISCOSITY ON THE CURRENT LAYERS EMERGING UPON PROPAGATION OF THE ALFVÉN PULSE IN A HYPERBOLIC MAGNETIC FIELD

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The propagation of the Alfvén pulse in the vicinity of the X-point in the presence of viscosity is studied for the first time. It is shown that, in contrast to the case of magnetosonic perturbation, where the dynamic viscosity η (the point is that we are dealing with dimensionless quantities), which is small compared to the magnetic plasma viscosity ν, does not affect the flow, this influence is of primary importance in the Alfvén case. The magnitude of the steady-state current density is proportional to $(\nu\eta)^{-1/4}$. It is also shown that at large times the distribution of the z-component of a magnetic field that is close to the distribution obtained in solving a linear problem is established in this significantly nonlinear problem. The effect of the heat conduction on this process is studied.

Introduction. It is known that the process of magnetic-field perturbation propagation undergoes significant changes when these perturbations approach the vicinity of the singularities of a magnetic field. In particular, current layers, in which the perturbation energy converts to the thermal and kinetic energy of the macroscopic plasma motion and the energy of fast particles, appear. The most interesting transformation of the perturbations occurs immediately near the zero lines (points and surfaces); in this connection, the studies [1–9] give results of the numerical and analytical investigations of the problem, which has the following mathematical formulation. Located in a hyperbolic magnetic field, the quiescent plasma occupied a square region. The motion was assumed to be two-dimensional ($\partial/\partial z = 0$). The waves generated by a remote source were modeled by specifying various perturbations of a magnetic field at the boundary of the region. The conditions that permit the plasma to flow in and out from this computational domain were established at its boundary.

The problem posed is studied in detail for the case of a magnetosonic pulse, when the motion is initiated by the perturbation of the z-component of the vector-potential of the magnetic field A at the boundary of the region. The results of calculations agree with analytical estimates and experimental data [1–7, 9]. In particular, the effect of the viscosity η on the processes of reconnection was discussed in [5, 10]. The analytical estimates presented in these studies and the unpublished results of the calculations performed with the use of the algorithms tested [9] for this problem make it possible to state that the dimensionless dynamic viscosity η, which is small compared to the dimensionless magnetic viscosity ν, almost does not affect the flow, and the z-component of the current density is equal to $j_z \sim \nu^{-1/2}$.

It is noteworthy that the majority of reconnection studies deal with flows in which the plasma velocity and the magnetic-field perturbations lie in the plane perpendicular to the zero line, i.e., the flows are of magnetosonic type. These flows occur in the most promising traps for thermonuclear fusion: tokamaks and stellarators. There are also studies in which three-dimensional MHD flows [11] and flows in which the plasma is described by kinetic equations [12] are modeled.

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In the space plasma, for example, in the plasma of the solar corona, flows in which there exist a velocity component and magnetic-field perturbations parallel to the zero line, i.e., Alfvén perturbations, play a great role. It was shown in [4, 13] that, in this case, the current layers along the separatrix surfaces separating independent magnetic fluxes can form. In contrast to the magnetosonic case, the current in these layers flows perpendicularly to the zero line along the separatrix surfaces. The specification of the z-component of the magnetic field H_z at the perturbation boundary corresponds to the propagation of the Alfvén pulse.

In this study, the mathematical formulation of the problem and results of the solution of the linear problem and the complete system of nonlinear equations are given.

1. Formulation of the Problem. In commonly accepted dimensionless variables, the initial equations of single-fluid magnetic hydrodynamics have the form [1, 6–9]

$$\begin{gathered}
\frac{\partial A}{\partial t}+(\boldsymbol{V}\nabla)A=\nu\Delta A, \qquad \frac{\partial H_z}{\partial t}+\operatorname{div}(\boldsymbol{V}H_z)=(\boldsymbol{H}\nabla)V_z+\nu\Delta H_z,\\
\rho\Big(\frac{\partial \boldsymbol{V}}{\partial t}+(\boldsymbol{V}\nabla)\boldsymbol{V}\Big)=-\nabla\Big(p+\frac{H_z^2}{2}\Big)-\Delta A\nabla A+\eta\Big(\Delta\boldsymbol{V}+\frac{1}{3}\nabla\operatorname{div}\boldsymbol{V}\Big),\\
\rho\Big(\frac{\partial V_z}{\partial t}+(\boldsymbol{V}\nabla)V_z\Big)=(\boldsymbol{H}\nabla)H_z+\eta\Delta V_z, \qquad \frac{\partial \rho}{\partial t}+\operatorname{div}(\rho\boldsymbol{V})=0,\\
\frac{1}{\gamma-1}\Big(\frac{\partial p}{\partial t}+\operatorname{div}(\boldsymbol{V}p)\Big)=\operatorname{div}(\chi\nabla T)-p\operatorname{div}(\boldsymbol{V})+\nu((\nabla H_z)^2+(\Delta A)^2)+\eta Q,\\
Q=2\Big(\Big(\frac{\partial V_x}{\partial x}\Big)^2+\Big(\frac{\partial V_y}{\partial y}\Big)^2\Big)+\Big(\frac{\partial V_x}{\partial y}+\frac{\partial V_y}{\partial x}\Big)^2+\Big(\frac{\partial V_z}{\partial x}\Big)^2+\Big(\frac{\partial V_z}{\partial y}\Big)^2-\frac{2}{3}(\operatorname{div}\boldsymbol{V})^2,\\
T=\frac{p}{\rho}, \qquad \boldsymbol{H}=(H_x,H_y)=\Big(\frac{\partial A}{\partial y},-\frac{\partial A}{\partial x}\Big), \qquad \boldsymbol{V}=(V_x,V_y), \qquad \gamma=\frac{5}{3}.
\end{gathered} \tag{1.1}$$

Here A is the z-component of the vector-potential of the magnetic field. The initial conditions correspond to the stationary solution of these equations: $A = A_0 = (x^2 - y^2)/2$ (X point), $H_z = 0$, $p = \beta$, $\rho = 1$, and $\boldsymbol{V} = 0$.

We dwell upon the boundary conditions. As in [4–8], for the velocities V_x and V_y at the boundary of the computational domain $-1 < x < 1$, $-1 < y < 1$ the conditions $\partial V_{x,y}/\partial \boldsymbol{n} = 0$ ($\boldsymbol{n}$ is the normal to the boundary) were chosen. In this work, for V_z, instead of the zero normal derivative as in [4–8], the gradient along a magnetic field was taken to be zero:

$$(\boldsymbol{H}\nabla)V_z = 0. \tag{1.2}$$

Condition (1.2) was also set for H_z at the lateral ($x = \pm 1$) boundaries. This is connected with the fact that, according to the meaning of the problem, the Alfvén wave arrives from infinity along the field $\boldsymbol{H}$.

The vector-potential A was assumed to be unchanged in time at the boundary; here the perturbation H_z for $y = \pm 1$ had the form

$$H_z(x, y = \pm 1) = \pm H_1 \min(t/t_s, 1), \tag{1.3}$$

i.e., H_z increased from zero to H_1 for t_s at the boundary.

In [4–8], the normal derivatives of p and ρ were set equal to zero if the plasma flows out from the computational domain through the given section of the boundary. Otherwise, p and ρ were assumed to be equal to their undisturbed values β and 1. This condition results in the following undesirable effect. At large times, an almost equilibrium configuration, in which the plasma velocity $\boldsymbol{V}$ is small and the pressure is much greater than the initial pressure, is formed. Therefore, when the direction of the velocity becomes opposite, the pressure changes abruptly at the boundary even for a small absolute value of the velocity. In view of this, for the quantities considered we imposed the conditions for the fluxes of these magnitudes, namely, if the velocity at the boundary is directed inward the computational domain, then $(\boldsymbol{V}\boldsymbol{n})p = (\boldsymbol{V}\boldsymbol{n})\beta$ and

$(\boldsymbol{Vn})\rho = (\boldsymbol{Vn})$. Otherwise, in the expressions $(\boldsymbol{Vn})p$ and $(\boldsymbol{Vn})\rho$, the values of p and ρ were assumed to be equal to their values in the near-boundary computational cell. Here the flux at the boundary, which is connected with dissipation in the pressure equation, was assumed to be zero: $\chi\nabla T = 0$; this allowed us to avoid the difficulties mentioned above.

Owing to the symmetry, the problem posed can be solved in a quarter of the domain. For numerical solution, an explicit first-order finite-difference scheme, which was treated in [9], was employed.

2. Linear Problem. We consider the propagation of the Alfvén pulse in a linear approximation, assuming that the poloidal field $\boldsymbol{H}$ does not depend on time, and H_z and V_z are described by the equations

$$\frac{\partial H_z}{\partial t} = (\boldsymbol{H}\nabla)V_z + \nu\Delta H_z, \qquad \rho\frac{\partial V_z}{\partial t} = (\boldsymbol{H}\nabla)H_z + \eta\Delta V_z \qquad (\rho = \mathrm{const}). \tag{2.1}$$

The boundary conditions have the form (1.2), (1.3).

It is clear that the stationary distribution of H_z is approximately the following:

$$H_z = H_1 \text{ for } y > |x|, \qquad H_z = -H_1 \text{ for } y < -|x|, \qquad H_z = 0 \text{ for } |x| > |y|. \tag{2.2}$$

The distribution of H_z in the neighborhood of the line $|x| = |y|$ will be smeared out under the influence of η and ν. After the replacement $V_z = \sqrt{\nu/\eta}V_{z*}$, one can see that the stationary solution of system (2.1) depends on only one parameter $\nu\eta$. Assuming that $\boldsymbol{H}\nabla \sim 1$ and $\Delta \sim l^{-2}$ (l is the layer thickness), we obtain the estimate $l \sim (\nu\eta)^{1/4}$ and, hence, $\boldsymbol{j}_\perp \sim H_1/l \sim H_1/(\nu\eta)^{1/4}$. For small $\nu\eta$, this dependence is supported by calculations. At the stationary stage, at large times, the poloidal current is maximal on the zero line (at the center of coordinates) and equals 9 for $\nu\eta = 5 \cdot 10^{-5}$ and 4.3 for $\nu\eta = 10^{-4}$. In this case, it was assumed that $H_1 = 0.5$.

Let us consider the case $\eta = 0$. According to (2.1), the stationary equation for V_z can have the solution only if H_z is constant along a magnetic field. Along the force lines that do not intersect the x axis, this condition can, in principle, be satisfied. However, the lines that intersect this axis connect the regions in which H_z has a different sign; as a result, this condition cannot be satisfied for them. Calculations show that, for $\eta = 0$, solution (2.1) tends to the discontinuous solution (2.2). Accordingly, the current increases unboundedly with time.

3. Nonlinear Problem. In the case of the complete system (1.1), the flow pattern is as follows. An Alfvén-type perturbation wave of the magnetic field H_z propagates from the boundaries $y = \pm 1$ to the zero line (the center of coordinates) along the force lines of the magnetic field. At distances from the center of coordinates at which the strength of the background poloidal field is comparable, in order of magnitude, with the strength of H_z in the wave ($r \sim H_1$), the flow becomes significantly nonlinear. Under the action of the magnetic pressure $H_z^2/2$, the plasma moves to the x axis, deforming the poloidal magnetic field. As a result, a layer of the current z-component elongated along the x axis appears (Fig. 1). A layer of poloidal current $\boldsymbol{j}_\perp = (-\partial H_z/\partial y, \partial H_z/\partial x)$ also appears, whose vector field is shown in Fig. 2. The values of j_z and $\boldsymbol{j}_\perp$ in these layers can be much greater than their values at the quasistationary stage; a situation where $|j_z| \gg |\boldsymbol{j}_\perp|$ can arise. At the quasistationary stage, we have $|\boldsymbol{j}_\perp| \ll |j_z|$. In these layers, the density and pressure of the plasma will also increase.

The current layers shown in Fig. 1 and 2 are greatly nonstationary. Owing to ohmic heating, the plasma pressure in the layer increases, and a gas-kinetic pressure wave, which propagates along a magnetic field to the boundaries $x = \pm 1$, arises. For $|y| > |x|$, the motion of this wave toward the boundaries $y = \pm 1$ is hindered by the field H_z whose pressure is high (of the order of H_1^2). As a result, at large times a quasistationary configuration in which $p + H_z^2/2 \approx \mathrm{const}$ is established. Figure 3 shows the distribution of $H_z^2/2$ at large times. Since now the excess pressure of the field H_z is compensated by the gas-kinetic pressure, rather than the deformation of the poloidal field, the current layer j_z disappears. We have $j_z \to 0$ as $t \to \infty$. The vector field $\boldsymbol{j}_\perp$ is shown in Fig. 4. The layer $\boldsymbol{j}_\perp$ is located along the lines $|y| = |x|$, and $|\boldsymbol{j}_\perp|$ reaches the maximum at the center of the region. The value of this maximum is denoted by j_∞. The velocity is $\boldsymbol{V} \to 0$ as $t \to \infty$.

Since $\boldsymbol{V}$ is small at the quasistationary stage and $A \approx A_0$, the distribution of V_z and H_z is close to that obtained in the solution of the linear system (2.1); it is determined by the parameters ν and η. At large

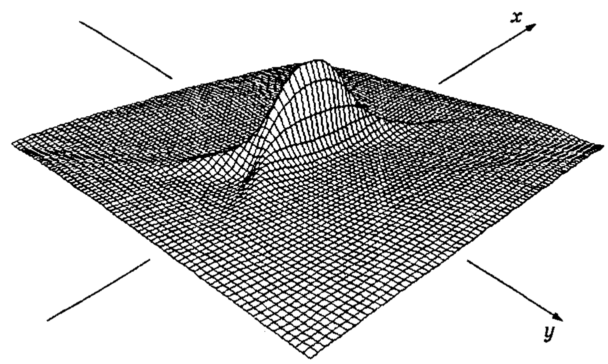

Fig. 1

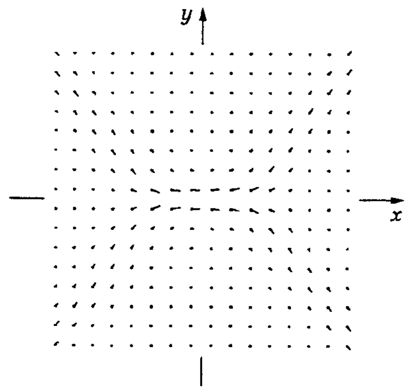

Fig. 2

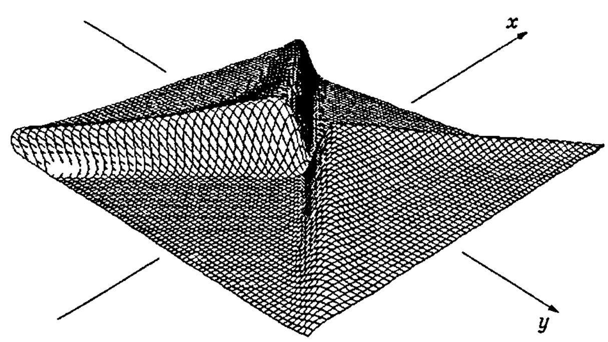

Fig. 3

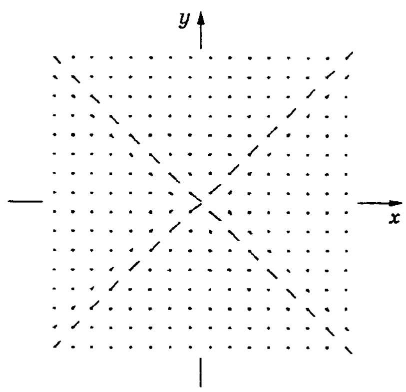

Fig. 4

times, the dependence of V_z and H_z on the parameters t_s (1.3), β, and χ is weak. The independence of the quasistationary mode on t_s is connected with the fact that we consider the solution of the problem for $t \gg t_s$. We note that the time of reaching a stationary regime by the solution is several hundred times the Alfvén time and depends not only on the product $\nu\eta$, but also on the relation between them. The weak effect of β is due to intense ohmic plasma heating. The plasma temperature grows even at the quasistationary stage and is much greater than the initial temperature. The weak dependence of the magnetic-field and pressure distributions on the thermal conduction is less obvious at large times. Here, since the effect of χ on the temperature is great, the density distribution should change accordingly. Figures 5 and 6 show the typical density distribution for $\chi \ll \nu$ and $\chi \geqslant \nu$, respectively. It is noteworthy that a similar phenomenon is also observed in the case of a magnetosonic pulse.

We give examples of some calculations. For $H_1 = 0.5$, $0.02 < \beta < 0.5$, $0.005\nu < \chi < 2\nu$, and $\nu = 0.005$ and $\eta = 0.001$ or $\nu = 0.001$ and $\eta = 0.005$, the calculated values of j_∞ vary from 8.6 to 9.1. For the same values, the solution of the linear problem (2.1) gives $j_\infty = 9$. For $\nu = \eta = 0.01$, the numerical calculations yield $j_\infty = 4.3$; this is in agreement with the solution of the linear problem. Thus, the distribution of the poloidal current in a quasistationary mode in the significantly nonlinear problem is close to its distribution in the linear case.

For small values of $\nu\eta$, from the calculations follows the dependence

$$j_\infty \approx CH_1(\nu\eta)^{-1/4}, \tag{3.1}$$

where $C \sim 100$.

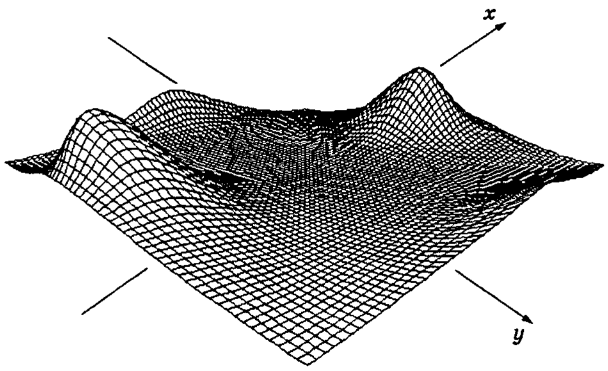

Fig. 5

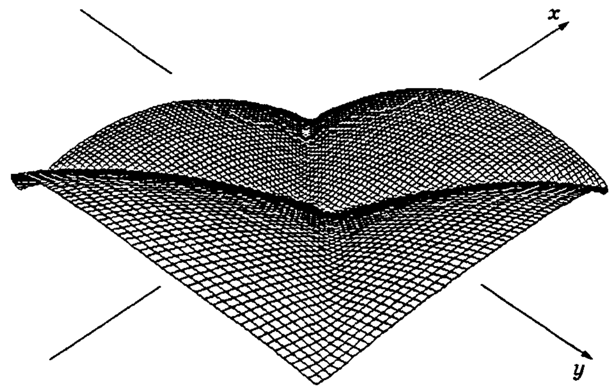

Fig. 6

In the case $\eta = 0$, for $\nu = 0.1$, $\chi = 0.02$, and $H_1 = 0.5$ the quantity $j_\perp$ exceeds 8. For the same values of ν and χ but for $\eta = 0.001$, the quantity $|j_\perp| \leqslant j_\infty \approx 4.3$, which corresponds to the dependence, (3.1). Note that, for $\eta = 0$, the distribution of the poloidal current does not reach a stationary distribution for any parameters of the problem. For example, for small thermal conduction, an intense plasma outflow from the computational domain is observed; as a result, the plasma falls off by several orders of magnitude. The current distribution here is far from stationary.

Conclusions. We have shown that the viscosity exerts a great effect on the value of the current in the problem considered. The distribution of H_z is close to that obtained in the solution of the linear problem, and the dependence $p + H_z^2/2 \approx \text{const}$ holds for the pressure profile.

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REFERENCES

1. K. V. Brushlinskii, A. M. Zaborov, and S. I. Syrovatskii, "Numerical analysis of the current layer in the vicinity of the zero line," *Fiz. Plazmy*, **6**, No. 2, 297–311 (1980).
2. *Neutral Current Layers in Plasma* [in Russian], Nauka, Moscow (1974). (*Tr. Fiz. Inst. Akad. Nauk SSSR*, **74**.)
3. *Flash Processes in Plasma* [in Russian], Nauka, Moscow (1979). (*Tr. Fiz. Inst. Akad. Nauk SSSR*, **110**.)
4. S. V. Bulanov, I. Ya. Butov, Yu. S. Gvaladze, et al., "Pinch discharges in plasma near the separatrix surfaces of magnetic fields," *Fiz. Plazmy*, **12**, No. 3, 309–327 (1986).

5. S. V. Bulanov, G. I. Dudnikova, V. P. Zhukov, et al., "Current layers in the vicinity of the critical points of the magnetic field," *Tr. Inst. Obshch. Fiz. Ross. Akad. Nauk*, **51**, No. 2, 101–123 (1996).
6. S. V. Bulanov, G. I. Dudnikova, V. P. Zhukov, et al., "Numerical modeling of the current layer in the vicinity of the zero line of the magnetic field," *Kratk. Soobshch. Fiz.*, Nos. 5 and 6, 28–33 (1994).
7. S. V. Bulanov, G. I. Dudnikova, V. P. Zhukov, et al., "Reconnection of the magnetic force lines in the vicinity of critical points," *Fiz. Plazmy*, **22**, No. 10, 867–895 (1996).
8. G. I. Dudnikova and V. P. Zhukov, "Formation of a current layer under the action of the Alfvén pulse in an ideally conducting plasma," *Vychisl. Tekhnol.*, **4**, No. 11, 108–118 (1995).
9. Yu. A. Berezin and G. I. Dudnikova, *Numerical Models of Plasma and Reconnection Processes* [in Russian], Nauka, Moscow (1985).
10. W. Park, D. Montinello, and R. White, "Reconnection rates of magnetic fields including the effects of viscosity," *Phys. Fluids*, **27**, No. 1, 134–149 (1984).
11. L. A. Charlton, J. A. Holmes, V. E. Lynch, and B. A. Carreras, "Compressible linear and nonlinear resistive MHD calculations in toroidal geometry," *J. Comput. Phys.*, **86**, 270–293 (1990).
12. L. C. Lee and D. Q. Ding, "Magnetic reconnection in a collisionless plasma," *Geophys. Res. Lett.*, **14**, No. 10, 1003–1006 (1987).
13. P. Rosenau, "Three-dimensional flow with neutral points," *Phys. Fluids*, **22**, 849 (1979).